Several candidate functions were considered, mainly from a family of smooth functions for the radius-dependent take-up stress σ_W. Linearly decaying functions for σ_W produced complicated integral expressions for which closed-form solutions were not obtained. In contrast, monotonically decaying take-up stress in the parabolic form produced a relatively simple integral expression. This type of parabolic function is shown below:

$$\sigma_W = \sigma_{WO}\left(1-\frac{\alpha}{2}\frac{r^2}{R^2}\right), \qquad (4.5)$$

where the parameter σ_{wo} is the initial value of take-up stress, and α defines the decay rate; $\alpha = 0$ for constant take-up tension, small values of α for slow decay, and larger values of α for rapid decay. Figure 8 depicts these three cases of decay rate. It is noted that monotonically decaying take-up tension can be achieved by, or is substantially equivalent to, using a bucket of water with a valve for the slow release of the water over time, thus decreasing the tension load over time.

For the family of take-up stress functions with parabolic decay, circumferential and radial stress components were found by substitution of Equation 4.5 into Equations 4.3 and 4.4:

$$\sigma_\theta = \sigma_{wo}\left\{1-\frac{\alpha r^2}{2R^2}+\frac{r_o^2-\beta r^2}{(2\beta r R)^2}\left[\alpha\beta(r^2-R^2)+(2\beta R^2-\alpha r_o^2)\ Ln\frac{\beta R^2-r_o^2}{\beta r^2-r_o^2}\right]\right\}, \qquad (4.5)$$

$$\sigma_r = \sigma_{wo}\frac{r_o^2-\beta r^2}{(2\beta r R)^2}\left[\alpha\beta(r^2-R^2)+(2\beta R^2-\alpha r_o^2)\ Ln\frac{\beta R^2-r_o^2}{\beta r^2-r_o^2}\right]. \qquad (4.6)$$

In the case of constant take-up tensile stress, when $\alpha = 0$,

$$\sigma_\theta = \sigma_{wo}\frac{3\beta r^2-r_o^2}{2\beta r^2}\ Ln\frac{\beta R^2-r_o^2}{\beta r^2-r_o^2}$$

and $$\sigma_r = \sigma_{wo} \frac{r_o^2 - \beta r^2}{2\beta r^2} \; Ln \frac{\beta R^2 - r_o^2}{\beta r^2 - r_o^2} \,. \tag{4.7}$$

Computations were made using the *Mathematica*® software and involved complex numbers,

$$Ln \frac{\beta R^2 - r_o^2}{\beta r^2 - r_o^2} \quad \rightarrow \; = A + iB$$

and stresses appeared to be expressed as follows: $\{\sigma_\theta, \sigma_r\} = Function\{D + iG\}$, where D and G are constants, and G is a negligibly small number. Thus, final expressions for stress components contained real numbers only.

Equations 4.7 were applied to the following case: core radius is 120mm, outer radius of the roll is 151.5mm, and constant wrapping stress is 1.38 x 10^5 kg/(mm s^2). Young's modulus of the tape material is 1.637 x 10^6 kg/(mm s^2) and Poisson's ratio is 0.4. Figures 9A-1, -2 and -3 show the distribution of the stresses in the roll obtained by using the second model with a rigid core; i.e. $\beta = -1$.

A comparison of the stress distribution computed using the first analytical model (a cylinder with tensile stresses) and the second analytical model (a shrunk ring) is shown in Figures 9B-1 and -2 for 10 (Figure 9B-1) and 2460 layers (Figure 9B-2). Curves for radial stresses almost coincide while the curves for circumferential stresses are very close to each other. The first model produced a curve with a parabolic shape and is located up to 0.7% above that obtained by using the second analytical model with β= -1. Additional computation for β= -2, which represents a very rigid core, showed a difference in circumferential stresses up to 4.4% between the first and the second models with the

maximum difference occurring near the core at $r = r_o$. A small difference in the radial stresses was also found at the beginning of the roll.

Figures 9C-1 and -2 show a comparison of the strain distribution obtained from the first and second models for 10 (Figure 9C-1) and 2460 (Figure 9C-2) layers of material using plane stress equations. As can be seen the curves obtained from the two models are very similar.

In conducting the above research it was needed to identify EFL as a function of several main parameters representing geometry, materials, and processing. During the research it was found that EFL is sensitive to the following parameters and factors:

Roll geometry – initial and final radius, $r_o < r < R$;

Stiffness of the reel core compared to that of buffer tube, β;

Young's modulus representing buffer tube material that depends on time and temperature, $E(t, T)$;

Poisson's ratio, ν;

Initial level of EFL, before reeling; and

Take-up stress function, including amplitude and decay rate α.

The EFL on the reel can be computed based on the value of EFL_o before reeling and strain in circumferential direction as:

$$EFL = EFL_o - \varepsilon_\theta \tag{4.8}$$

that is:

$$EFL = EFL_o - \frac{\sigma_{wo}}{E(t,T^o)}\ \frac{(1+\nu)}{4}\ \left\{\left[(1-2\nu)+\frac{r_o^{\,2}}{\beta r^2}\right]\left(\frac{\alpha r_o^{\,2}}{\beta R^2}-2\right)\ Ln\frac{\beta R^2 - r_o^2}{\beta r^2 - r_o^2}\ +\right.$$